Math 123: Series

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Outline

Must Know Theorems Regarding Limits

1. $lim_{n\to\infty}\frac{ln(n)}{n} = 0$
2. $lim_{n\to\infty}n^{\frac{1}{n}} = 1$
3. $lim_{n\to\infty}x^{\frac{1}{n}} = 1$ if $x > 0$
4. $lim_{n\to\infty}x^n = 0$ if $|x| < 1$
5. $lim_{n\to\infty}(1 + \frac{x}{n})^n = e^x$
6. $lim_{n\to\infty}\frac{x^n}{n!} = 0$

Series in terms of Sequences

Roughly, an infinite series $\Sigma_{i=1}^{\infty} a_i$ denotes the sum of the terms in the sequence $\{a_i\}_{i=1}^{\infty}$.

Definition

The **n-th partial sum** for a sequence $\{a_i\}_{i=1}^{\infty}$ is

$$S_n = \Sigma_{i=1}^{n} a_i = a_1 + a_2 + a_3 + ... + a_n$$

Definition

A **Series**

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Exercise: Given a constant r find $\Sigma_{i=0}^{\infty} r^i$ when it exists.
Exercise: Use partial sums to find $\Sigma_{i=1}^{\infty} \frac{1}{i^2+i}$.

First tests for convergence

Theorem

If a series $\Sigma_{i=1}^{\infty} a_i$ *converges then* $lim_{n\to\infty} a_i = 0$.

Theorem

If $lim_{n\to\infty} a_i \neq 0$ *or does not exist, then* $\Sigma_{i=1}^{\infty} a_i$ *does not converge.*

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Exercise: Determine the convergence or divergence of $\Sigma_{i=1}^{\infty} ln(\frac{i^2+1}{2i^2+1})$

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Exercise: Determine the convergence or divergence of $\Sigma_{i=1}^{\infty} \frac{e^i}{i^2}$.

Integral Test

Theorem

Let f be a continuous, positive, decreasing function on $[c, \infty]$. If $a_i = f(i)$, then

1. *If $\int_c^\infty f(x)dx$ is convergent, then $\Sigma_{i=c}^\infty a_i$ is convergent.*
2. *If $\int_c^\infty f(x)dx$ is divergent, then $\Sigma_{i=c}^\infty a_i$ is divergent.*

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Show that $\Sigma_{i=1}^\infty \frac{1}{i^p}$ is convergent for $p > 1$.

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Show that $\Sigma_{i=1}^\infty \frac{1}{i^p}$ is convergent for $p > 1$.
Determine if $\Sigma_{i=2}^\infty \frac{1}{i ln(i)}$ is convergent or divergent.

The Direct Comparison Test

Theorem

Let $\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ be positive series.

1. *If $a_i \leq b_i$ for all i and $\sum_{i=1}^{\infty} b_i$ converges, then $\sum_{i=1}^{\infty} a_i$ converges.*
2. *If $a_i \leq b_i$ for all i and $\sum_{i=1}^{\infty} a_i$ diverges, then $\sum_{i=1}^{\infty} b_i$ diverges.*

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Determine if $\sum_{i=1}^{\infty} \frac{1}{i!}$ is convergent or divergent.

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Determine if $\Sigma_{i=1}^{\infty} \frac{1}{i!}$ is convergent or divergent.

Determine if $\Sigma_{i=2}^{\infty} \frac{\sqrt{i}}{2i-1}$ is convergent or divergent.

Limit Comparison Test

Theorem

Let $\Sigma_{i=1}^{\infty} a_i$ and $\Sigma_{i=1}^{\infty} b_i$ be positive series. If

$$lim_{i\to\infty}\frac{a_i}{b_i} = C$$

where C is a finite positive constant, then either both $\Sigma_{i=1}^{\infty} a_i$ and $\Sigma_{i=1}^{\infty} b_i$ converge or both $\Sigma_{i=1}^{\infty} a_i$ and $\Sigma_{i=1}^{\infty} b_i$ diverge.

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Determine if $\Sigma_{i=1}^{\infty} \frac{i+2}{(i+1)^3}$ is convergent or divergent.

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Determine if $\Sigma_{i=1}^{\infty} \frac{i+2}{(i+1)^3}$ is convergent or divergent.

Determine if $\Sigma_{i=1}^{\infty} \frac{2i^2-1}{i^2 3^i}$ is convergent or divergent.