Math 123: Introduction to Differential Equations

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Outline

Differential equations

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Example: Solve $y'' = -y$

The Math of Frisky Bunnies

Suppose bunnies reproduce according to the following rules

1. We start in month zero with one male and one female bunny.
2. Every month each mature female bunny gives birth to one male and one female bunny.
3. After being born, each bunny takes one month to become mature.

Let $B(t)$ be the number of bunnies t months after month zero.

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Estimate $B'(t)$.
How should we model $B(t)$?

A Few Famous Differential Equations

1. Einstein's field equation in general relativity
2. The Navier-Stokes equations in fluid dynamics
3. Verhulst equation - biological population growth
4. The Black-Scholes PDE - models financial markets

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Example: Solve the following D.E. $\frac{dy}{dx} = \frac{cos(x)}{y^2}$.
Example: Solve the following D.E. $\frac{dy}{dx} = y$.

Logistic Growth (the Verhulst model)

Hypotheses for the population model:

1. For small populations the population growth is proportional to the population size.
2. The population can not grow larger than a carrying capacity M.

Where k and M are constants.

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Exercise: Find the general solution to this D.E.