Math 123: First Order D.E.s and Slope Fields

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Outline

1. First Order Differential Equations

2. Slope Fields

Types of Differential equations

Definition

A differential equation is any equation involving a function, its derivatives.

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If the n-th derivative is the largest derivative that appears in the differential equation, we say it is an **nth order** differential equation.

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A first order differential equation $\frac{dy}{dx} = F(x, y)$ together with and **initial condition** $y(a) = b$ is an **initial value problem**

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Example: Solve the initial value problem $y' = \frac{1}{2}y$ and $y(0) = 2$

Example: Solve the initial value problem $\frac{dy}{dx} = \frac{xe^x}{cos(y)}$ and $y(0) = 0$

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Example: Find $lim_{x \to \infty} y(x)$ if $y(x)$ is a solution to the IVP $y' = (y - 1)(y - 3)$ and $y(0) = 0$.

Slope Fields Using Dfield

Here we will be using the free internet software Dfield.

Example: Determine the limits as x goes to infinity for solutions to $y' = (\frac{1}{2}y(5 - y)$ (A Verhulst Equation).

Example: Determine the initial values for which solutions to $y' = x^2 + y^2 - 4$ are **always** increasing.