Math 123: Constant Coefficient 2nd Order Homogeneous Linear D.E.s

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Outline

Motivation

Our goal is to solve constant-coefficient, linear, 2nd-order, homogeneous differential equations.

Given a linear 2nd order homogeneous **constant-coefficient** differential equation

$$ay'' + by' + cy = 0,$$

the **Auxiliary Equation** is

$$am^2 + bm + c = 0.$$

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The roots of the auxiliary equation determines the general solution.

Case 1: Distinct Roots

If $am^2 + bm + c$ has distinct roots m_1 and m_2, then the general solution to $ay'' + by' + cy = 0$ is

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

.

Case 2: Repeated Roots

If $am^2 + bm + c$ has a repeated root m_1, then the general solution to $ay'' + by' + cy = 0$ is

$$y = c_1 e^{m_1 x} + c_2 x e^{m_1 x}$$

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Magic!

$$e^{i\theta} = cos(\theta) + isin(\theta)$$

Case 3: Complex Roots

If $am^2 + bm + c$ has complex roots $m_1 = \alpha + i\beta$ and $m_2 = \alpha - i\beta$, then the general solution to $ay'' + by' + cy = 0$ is

$$y = c_1 e^{\alpha x} cos(\beta x) + c_2 e^{\alpha x} sin(\beta x)$$

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