

Math 123: First Order D.E.s and Slope Fields

Ryan Blair

CSU Long Beach

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Outline

1 First Order Differential Equations

2 Slope Fields

Types of Differential equations

Definition

A differential equation is any equation involving a function, its derivatives.

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If the n -th derivative is the largest derivative that appears in the differential equation, we say it is an **n th order** differential equation.

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Example: Solve the initial value problem $y' = \frac{1}{2}y$ and $y(0) = 2$

Example: Solve the initial value problem $\frac{dy}{dx} = \frac{xe^x}{\cos(y)}$ and $y(0) = 0$

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Example: Find $\lim_{x \rightarrow \infty} y(x)$ if $y(x)$ is a solution to the IVP $y' = (y - 1)(y - 3)$ and $y(0) = 0$.

Slope Fields Using Dfield

Here we will be using the free internet software Dfield.

Example: Determine the limits as x goes to infinity for solutions to $y' = (\frac{1}{2}y(5 - y))$ (A Verhulst Equation).

Example: Determine the initial values for which solutions to $y' = x^2 + y^2 - 4$ are **always** increasing.