Math 123: Trig Integrals and Trig Substitution

Ryan Blair

CSU Long Beach

Tuesday January 26, 2016

Outline

How do we integrate Trigonometric functions?

Example: $\int sin(x)dx$ and $\int cos(x)dx$.

How do we integrate Trigonometric functions?

Example: $\int sin(x)dx$ and $\int cos(x)dx$.
Example: $\int sin^3(x)dx$

How do we integrate Trigonometric functions?

Example: $\int sin(x)dx$ and $\int cos(x)dx$.
Example: $\int sin^3(x)dx$

For $\int cos^{odd}(x)dx$ or $\int sin^{odd}(x)dx$ use

$$cos^2(x) + sin^2(x) = 1$$

and u-substitution

How do we integrate Trigonometric functions?

Example: $\int sin(x)dx$ and $\int cos(x)dx$.
Example: $\int sin^3(x)dx$

For $\int cos^{odd}(x)dx$ or $\int sin^{odd}(x)dx$ use

$$cos^2(x) + sin^2(x) = 1$$

and u-substitution

Example: $\int cos^5(x)dx$

How do we integrate Trigonometric functions?

Example: $\int sin^2(x)dx$

How do we integrate Trigonometric functions?

Example: $\int sin^2(x)dx$

For $\int cos^{even}(x)dx$ or $\int sin^{even}(x)dx$ use

$$cos^2(x) = \frac{1}{2}(1 + cos(2x))$$

$$sin^2(x) = \frac{1}{2}(1 - cos(2x))$$

possibly multiple times

How do we integrate Trigonometric functions?

Example: $\int sin^2(x)dx$

For $\int cos^{even}(x)dx$ or $\int sin^{even}(x)dx$ use

$$cos^2(x) = \frac{1}{2}(1 + cos(2x))$$

$$sin^2(x) = \frac{1}{2}(1 - cos(2x))$$

possibly multiple times

Example: $\int cos^4(x)dx$

How do we integrate Trigonometric functions?

Example: $\int sin^2(x)cos^3(x)dx$.

How do we integrate Trigonometric functions?

Example: $\int sin^2(x)cos^3(x)dx$.

For $\int sin^{anything}(x)cos^{odd}(x)dx$ or $\int cos^{anything}(x)sin^{odd}(x)dx$ use

$$cos^2(x) + sin^2(x) = 1$$

and u-substitution

How do we integrate Trigonometric functions?

Example: $\int sin^2(x)cos^3(x)dx$.

For $\int sin^{anything}(x)cos^{odd}(x)dx$ or $\int cos^{anything}(x)sin^{odd}(x)dx$ use

$$cos^2(x) + sin^2(x) = 1$$

and u-substitution
Question: What about $\int sin^{even}(x)cos^{even}(x)dx$

How do we integrate Trigonometric functions?

Example: $\int tan(x)sec^4(x)dx$.

How do we integrate Trigonometric functions?

Example: $\int tan(x)sec^4(x)dx$.

For integrals involving $tan(x)$ and $sec(x)$ use

$$1 + tan^2(x) = sec^2(x)$$

and u-substitution.

Some Challenges

Example: Find $\int sec(x)dx$.
Example: Find $\int sec^3(x)dx$.

A Motivating Example

Find $\int_{-1}^{1} \sqrt{1 - x^2} dx$ in two different ways.

Method One: Geometric.

Method Two: Using Trigonometric identities.

A Motivating Example

Find $\int_{-1}^{1} \sqrt{1-x^2}dx$ in two different ways.

Method One: Geometric. Since $y = \sqrt{1-x^2}$ is the top half of the unit circle, use definition of integral as area under the curve.

Method Two: Using Trigonometric identities.

A Motivating Example

Find $\int_{-1}^{1} \sqrt{1-x^2}dx$ in two different ways.

Method One: Geometric. Since $y = \sqrt{1-x^2}$ is the top half of the unit circle, use definition of integral as area under the curve.

Method Two: Using Trigonometric identities. Make the substitution $x = sin(\theta)$ and use $cos^2(\theta) + sin^2(\theta) = 1$.

Trig Substitution

For integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$ or $\sqrt{x^2 + a^2}$ where a is a constant, we can often integrate by constructing a right triangle with one of these values as a side length.

Trig Substitution

For integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$ or $\sqrt{x^2 + a^2}$ where a is a constant, we can often integrate by constructing a right triangle with one of these values as a side length.

Method Three: Find $\int_{-1}^{1} \sqrt{1 - x^2}dx$ by building the relevant right triangle and making a substitution.

Trig Substitution

For integrals involving $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$ or $\sqrt{x^2 + a^2}$ where a is a constant, we can often integrate by constructing a right triangle with one of these values as a side length.

Method Three: Find $\int_{-1}^{1} \sqrt{1 - x^2}dx$ by building the relevant right triangle and making a substitution.

Example: Find $\int \frac{1}{x^2\sqrt{x^2+9}}$

Example: Find $\int \frac{1}{\sqrt{x^2-4}}$