Math 123: Approximate Integration

Ryan Blair

CSU Long Beach

Tuesday February 2, 2016

Outline

Steps of Partial Fraction Expansion

When $p(x)$ and $q(x)$ are polynomials, we want to find

$$\int \frac{p(x)}{q(x)}dx$$

Steps of Partial Fraction Expansion

When $p(x)$ and $q(x)$ are polynomials, we want to find

$$\int \frac{p(x)}{q(x)} dx$$

Step 1: If $deg(p(x)) \geq deg(q(x))$, then divide.

Steps of Partial Fraction Expansion

When $p(x)$ and $q(x)$ are polynomials, we want to find

$$\int \frac{p(x)}{q(x)} dx$$

Step 1: If $deg(p(x)) \geq deg(q(x))$, then divide.

Step 2: Factor the denominator (sometimes this is quite hard)

Steps of Partial Fraction Expansion

When $p(x)$ and $q(x)$ are polynomials, we want to find

$$\int \frac{p(x)}{q(x)} dx$$

Step 1: If $deg(p(x)) \geq deg(q(x))$, then divide.

Step 2: Factor the denominator (sometimes this is quite hard)
Example Find $\int \frac{1}{x^3 - x} dx$

When the Denominator has all Distinct Linear Factors

Step 3: Depends on the factorization
Recall we are interested in evaluating $\int \frac{p(x)}{q(x)}$

When the Denominator has all Distinct Linear Factors

Step 3: Depends on the factorization
Recall we are interested in evaluating $\int \frac{p(x)}{q(x)}$
Case 1: $q(x)$ is the product of distinct linear factors

$$q(x) = (a_1x + b_1)(a_2x + b_1)...(a_kx + b_k)$$

In this case we let

$$\frac{p(x)}{q(x)} = \frac{A_1}{(a_1x + b_1)} + \frac{A_2}{(a_2x + b_1)} + ... + \frac{A_k}{(a_kx + b_k)}$$

and we solve algebraically for $A_1, A_2, ..., A_k$.
Example Find $\int \frac{1}{x^3 - x} dx$

Definition

(**Definite Integral**)If f is a function defined for $a \leq x \leq b$, we divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = \frac{b-a}{n}$. We let $x_0(= a), x_1, x_2, ..., x_n(= b)$ be the endpoints of these subintervals and we let $x_1^*, x_2^*, ..., x_n^*$ be any **sample points** in these subintervals. Then the **definite integral of** f **from** a **to** b is

$$\int_a^b f(x)dx = lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

provided that this limit exists. If it does exist, we say that f is **integrable** on $[a, b]$.

Picking Sample points and Approx. Integrals

$\int_a^b f(x)dx$ is approximated by each of the following

$$R_n = f(x_1)\Delta x + f(x_2)\Delta x + ... + f(x_n)\Delta x$$

$$L_n = f(x_0)\Delta x + f(x_1)\Delta x + ... + f(x_{n-1})\Delta x$$

$$M_n = f(\frac{1}{2}(x_0 + x_1))\Delta x + f(\frac{1}{2}(x_1 + x_2))\Delta x + ... + f(\frac{1}{2}(x_{n-1} + x_n))\Delta x$$

Picking Sample points and Approx. Integrals

$\int_a^b f(x)dx$ is approximated by each of the following

$$R_n = f(x_1)\Delta x + f(x_2)\Delta x + ... + f(x_n)\Delta x$$

$$L_n = f(x_0)\Delta x + f(x_1)\Delta x + ... + f(x_{n-1})\Delta x$$

$$M_n = f(\frac{1}{2}(x_0 + x_1))\Delta x + f(\frac{1}{2}(x_1 + x_2))\Delta x + ... + f(\frac{1}{2}(x_{n-1} + x_n))\Delta x$$

Example Find R_4 for $\int_0^1 x^2 dx$ (is this an under estimate or an over estimate?)

Picking Sample points and Approx. Integrals

$\int_a^b f(x)dx$ is approximated by each of the following

$$R_n = f(x_1)\Delta x + f(x_2)\Delta x + ... + f(x_n)\Delta x$$

$$L_n = f(x_0)\Delta x + f(x_1)\Delta x + ... + f(x_{n-1})\Delta x$$

$$M_n = f(\frac{1}{2}(x_0 + x_1))\Delta x + f(\frac{1}{2}(x_1 + x_2))\Delta x + ... + f(\frac{1}{2}(x_{n-1} + x_n))\Delta x$$

Example Find R_4 for $\int_0^1 x^2 dx$ (is this an under estimate or an over estimate?)

Example Find L_4 for $\int_0^1 x^2 dx$ (is this an under estimate or an over estimate?)

Picking Sample points and Approx. Integrals

$\int_a^b f(x)dx$ is approximated by each of the following

$$R_n = f(x_1)\Delta x + f(x_2)\Delta x + ... + f(x_n)\Delta x$$

$$L_n = f(x_0)\Delta x + f(x_1)\Delta x + ... + f(x_{n-1})\Delta x$$

$$M_n = f(\frac{1}{2}(x_0 + x_1))\Delta x + f(\frac{1}{2}(x_1 + x_2))\Delta x + ... + f(\frac{1}{2}(x_{n-1} + x_n))\Delta x$$

Example Find R_4 for $\int_0^1 x^2 dx$ (is this an under estimate or an over estimate?)

Example Find L_4 for $\int_0^1 x^2 dx$ (is this an under estimate or an over estimate?)

Example Find M_4 for $\int_1^2 e^{x^2} dx$ (is this an under estimate or an over estimate?)

Approximating by Trapezoids

Recall that the area of a trapezoid with parallel sides of length a and b and of height h is

$$A = \frac{1}{2}(a+b)h$$

When we use trapezoids to approximate the area under the curve, we get

$$T_n = \frac{1}{2}\Delta x(f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n))$$

Approximating by Trapezoids

Recall that the area of a trapezoid with parallel sides of length a and b and of height h is

$$A = \frac{1}{2}(a+b)h$$

When we use trapezoids to approximate the area under the curve, we get

$$T_n = \frac{1}{2}\Delta x(f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n))$$

Example Show T_n is the average of L_n and R_n

Approximating by Trapezoids

Recall that the area of a trapezoid with parallel sides of length a and b and of height h is

$$A = \frac{1}{2}(a+b)h$$

When we use trapezoids to approximate the area under the curve, we get

$$T_n = \frac{1}{2}\Delta x(f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n))$$

Example Show T_n is the average of L_n and R_n
Example Find T_4 for $\int_0^1 x^2 dx$