Math 123: Linear D.E.s of First and Second Order

Ryan Blair

CSU Long Beach

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Outline

1. First Order Differential Equations
2. Integrating Factor Method
3. Superposition Principle

Types of Differential equations

Definition

A first order **linear** D.E. is of the form

$$y' + Q(x)y = R(x)$$

where $Q(x)$ and $R(x)$ are functions of x.

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A second order **linear** D.E. is of the form

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where $P(x)$, $Q(x)$ and $R(x)$ are functions of x.

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where $P(x)$, $Q(x)$ and $R(x)$ are functions of x.

If $R(x) = 0$ we call the D.E. **homogeneous**.

Integrating Factor Method

Question 1: Given a D.E. $y' + Q(x)y = R(x)$, if you could find a function $f(x)$ such that

$$\frac{d(f(x)y)}{dx} = f(x)(y' + Q(x)y)$$

could you solve the D.E.?

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Question 2: Given a D.E. $y' + Q(x)y = R(x)$, can you find the formula for a function $f(x)$ such that

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Answer: Yes! $f(x) = e^{\int Q(x)dx}$

Integrating Factor Method

To solve $y' + Q(x)y = R(x)$,

1. Multiply both sides by $f(x) = e^{\int Q(x)dx}$
2. Recognize that the L.H.S. is $\frac{d(f(x)y)}{dx}$
3. Integrate both sides and solve for y.

Exercise: Solve $y' + 2y = 2e^x$.
Exercise: Solve $xy' + y = \sqrt{x}$.

Superposition Principle

Theorem

Given a homogeneous linear differential equation with solutions $f(x)$ and $g(x)$ then $a \cdot f(x) + b \cdot g(x)$ is also a solution for any constants a and b.

Exercise: Demonstrate this theorem for the D.E.
$y'' + P(x)y' + Q(x)y = 0$