Math 123: Series Convergence Tests III

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CSULB

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Conditional and Absolute Convergence

Definition

A series $\Sigma_{i=1}^{\infty} a_i$ is **absolutely** convergent if $\Sigma_{i=1}^{\infty} |a_i|$ is convergent.

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A series $\Sigma_{i=1}^{\infty} a_i$ is **conditionally** convergent if $\Sigma_{i=1}^{\infty} |a_i|$ is divergent and $\Sigma_{i=1}^{\infty} a_i$ is convergent.

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Determine conditional or absolute convergence of $\Sigma_{i=1}^{\infty} \frac{cos(i)}{i^2}$.

Ratio Test

Theorem

Given a series $\sum_{i=1}^{\infty} a_i$*. If*

$$lim_{i\to\infty}\left|\frac{a_{i+1}}{a_i}\right| = L,$$

then

1. *If* $L < 1$*, the series converges absolutely.*
2. *If* $L = 1$*, the test is inconclusive.*
3. *If* $L > 1$*, the series diverges.*

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Helps with the crazy stuff: $\Sigma_{i=1}^{\infty} \frac{i^i}{i!}$

Root Test

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Given a series $\sum_{i=1}^{\infty} a_i$*. If*

$$lim_{i \to \infty} |a_i|^{\frac{1}{i}} = L,$$

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Helps with series involving variable powers: $\Sigma_{i=1}^{\infty} i(\frac{2}{3})^i$