MATH 233, HOMEWORK 3

GROUPS AND METRIC SPACES

Due by 10 am, Friday, Feb. 15st

1. Homework Policy

You are strongly encouraged to work in groups to exchange ideas and help each other understand how to approach problems, but the work you turn in must be your own! If you work with others on an assignment, be sure to indicate the names of the other students on your homework. Additionally, if you use any outside resources (i.e. internet sources, other mathematicians, other books) to help you solve homework problems, you must cite your sources. Failure to follow these rules will result in a score of zero on an assignment and may constitute a violation of academic integrity.

Homework must be legible, well-organized, and written in complete sentences. Handwritten work is fine, but you are encouraged to type up the problems in LaTeX.

Additional guidelines: If you submit hand written work make sure it is written legibly and stapled. If you submit work through email mail, it must me submitted as a **single pdf file** and have your name on the first page. Failure to follow these guidelines with result in a loss of points.

2. Readings and Responses.

(1) Reread Section 2.1.
(2) Read Section 2.2.
(3) What is the definition of metric space? What is the point of the definition?

3. Problems

(1) Write a complete proof of Theorem 2.1.6 to the best of your ability.
(2) Let $\mathcal{A}$ be the set of all continuous functions from $\mathbb{R}$ to $\mathbb{R}$ together with the operation of function addition (i.e. $f(x) \circ g(x) = f(x) + g(x)$). What function must you pick as the identity to make this set and operation a group (Remember that to define a function you have to define its domain and range and where it takes each point in its domain)? Prove that $\mathcal{A}$ with the identity you defined and the operation of function addition satisfy the (G2) axiom.
(3) Let $\{0, 1, 2\}$ be a set with identity 0 and operation $\circ$ defined by $0 \circ 0 = 0$, $0 \circ 1 = 1$, $0 \circ 2 = 2$, $1 \circ 0 = 1$, $1 \circ 1 = 2$, $1 \circ 2 = 0$, $2 \circ 0 = 2$, $2 \circ 1 = 0$, $2 \circ 2 = 1$. Show that this set, identity and operation satisfy the (G3) axiom.
(4) Do part 1. of example 2.2.4
(5) Write a complete proof of Theorem 2.2.5. You may copy verbatim the parts of the proof given in the textbook (no need for quotation marks) and should fill in the rest.