Midterm Exam 2

Math 500, Fall 2011

INSTRUCTIONS:

- This exam is due at the start of class on Thursday, November 17.
- You may use your textbook (Munkres), your own homework assignments, your class notes, and first midterm, but NO OTHER SOURCES. No other books, no discussing with other people, no internet sources.
- Abide by Penn's Code of Academic Integrity. If you have any questions or concerns, let me know.
- If you have questions about the exam or if you think there is a mistake, please email me (ryblair@math.upenn.edu).
- Write legibly, in COMPLETE SENTENCES, and explain your work carefully. Please write up your solutions very clearly and try to give an "appropriate" level of detail. If there is any doubt about how much detail to give, or about what you may quote without proof, just ask me by e-mail. **If you have been asked to type Midterm 2 please make sure to do so.**
- You may quote results from class and from Munkres that we have discussed. (You don't need to re-prove anything we have already done.)
- Typing the solutions is fine, if you prefer.
- Good luck!

Name ________________________________

Total Score __________ (/ 60 points)

1. *(10 points)* Let $X = \{(x, y) \in \mathbb{R}^2 | x^2 + y^2 \leq 1\}$. Let $\sim$ be the equivalence relation on X such that

$$(x, y) \sim (x, y) \;\; for \;\; x^2 + y^2 < 1$$

$$(x_1, y_1) \sim (x_2, y_2) \;\; for \;\; x_1^2 + y_1^2 = x_2^2 + y_2^2 = 1$$

Carefully show $X/\sim$ is homeomorphic to S^2.

2. *(10 points)* Mark each of the following statements true or false. If you mark a statement as false provide a counter example. No additional justification is necessary.

   (a) If X is a manifold, then X is path connected.

   (b) If X is connected and first-countable, then X is Hausdorff.

   (c) If $f : X \to Y$ is a continuous, surjective, open map, then f is a quotient map.

   (d) If A is a closed and bounded subset of a second countable metric space, then A is compact.

3. *(10 points)* Let X and Y be topological spaces. Let $\pi_1 : X \times Y \to X$ be the projection map $\pi_1(x \times y) = x$. Prove that if Y is compact, then π_1 is a closed map. (Hint: if C is a closed subset of $X \times Y$, consider separately the cases in which $\pi_1(C) = X$ and $\pi_1(C) \neq X$. For the latter, try using the tube lemma.)

4. *(10 points)* Let $p : X \to Y$ be a quotient map. Show that if $p^{-1}(\{y\})$ is connected for each $y \in Y$ and Y is connected, then X is connected.

5. *(10 points)* Assume that $[0, \pi)$ is uncountable. Show that if A is countable subset of $\mathbb{R}^2$, then $\mathbb{R}^2 - A$ is path connected. (Hint: How many lines are there passing through a given point in $\mathbb{R}^2$.

6. *(10 points)* Using a "cut-and-paste" argument similar to the one used in class to demonstrate that $\mathbb{RP}^2$ is homeomorphic to the disk glued to the Mobius band along their S^1 boundary, show that the following quotient space is homeomorphic to two Mobius bands glued together along their S^1 boundary. Hint: Use colors and pay attention to the orientations on the identified edges.