MATH 550A, HOMEWORK 1

BASIS; ORDER, PRODUCT, AND SUBSPACE TOPOLOGIES

Due at start of class, Tuesday, 2/4

Reading. Read §13–16 of Munkres.
Exercises (to do on your own).

(1) If X is a set, show that the collection of all one-point sets $\{x\}$ is a basis for the discrete topology.
(2) Check that the given basis for $\mathbb{R}_\ell$ is in fact a basis for a topology. Do same for $\mathbb{R}_K$.
(3) Prove that both $\mathbb{R}_\ell$ and $\mathbb{R}_K$ are *strictly* finer than $\mathbb{R}$ (with the standard topology) by finding open sets for $\mathbb{R}_\ell$ and $\mathbb{R}_K$ that are not open in $\mathbb{R}$.
(4) Prove that $\mathbb{R}_\ell$ and $\mathbb{R}_K$ are not comparable to each other.
(5) Munkres §16 exercise 3.
(6) Explain why the set $\{1/2\} \times (1/2, 1]$ is *not* open in the ordered square (c.f. Example 3 of §16).

Problems (to turn in).

(1) Munkres §13 exercise 8. (You may use standard facts about the real and rational numbers.)
(2) Munkres §16 exercise 1.
(3) Munkres §16 exercise 4.
(4) Munkres §16 exercise 9. (Hint: show that a basic open set for one topology is an open set in the other topology, and vice versa. Why is this enough? For the last part, use exercise 5a from this section of Munkres.)