

# Math 550A, Homework 3

## Continuous Functions

Due in class, Thursday, 2/20

**Reading** Read §18 of Munkres.

### Exercises (to do on your own)

1. Show that the subspace  $(a, b) \subset \mathbb{R}$  with  $a < b$  is homeomorphic to  $(0, 1)$ .
2. Define  $S^1$  to be the following subset of  $\mathbb{R}^2$ :

$$S^1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}.$$

Prove that  $S^1$  is closed by using the fact that the function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ , where  $f(x, y) = x^2 + y^2$ , is continuous.

3. (Infinite pasting lemma?) Suppose  $\{A_\alpha\}$  is a collection of closed subsets of  $X$  whose union is  $X$  and  $f : X \rightarrow Y$  is a map such that every restriction map  $f|_{A_\alpha} : A_\alpha \rightarrow Y$  is continuous. Must  $f$  be continuous?

### Problems (to turn in)

1. Munkres §18, exercise 1.
2. Munkres §18, exercise 8 (replace  $Y$  with  $\mathbb{R}$  if you would like).
3. Munkres §18, exercise 11. Be sure to also read exercise 12.