Math 550A, Homework 8

Separation Axioms

Due in class, Thursday, 4/24

Reading §30

Exercises (to do on your own)

1. Prove or disprove that $\mathbb{R}^\omega$ with the uniform topology is separable. Now do the same for Lindelöf, first-countable and second-countable.
2. Munkres, §30, exercise 11.
3. Munkres, §30, exercise 12.
4. Munkres, §30, exercise 14.

Problems (to turn in)

1. Munkres §30, exercise 5. (Together with what we showed in class, this proves that for metrizable spaces, the notions of second countable, Lindelöf, and separable all coincide.)
2. A space X is said to have *unique limits* if every convergent sequence $\{x_n\}$ has a unique limit. (i.e., $x_n \to x$ and $x_n \to y$ imply that $x = y$). Prove that if X has unique limits and is first countable, then X is Hausdorff.
3. Munkres, §30, exercise 4.