Announcements

- HW 4 due date posponed to Tuesday
- Midterms will be graded & returned by Tuesday

Outline

- Review of last time
- Brower fixed pt theorem

Last time

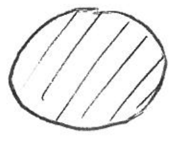
Def If $A < X$, a <u>retraction</u> of X to A is a continuous map $r : X \to A$ s.t. $r|_A = id_A$.

Lemma 55.1 If there is a retraction of X onto A, then ~~there is a~~ the homomorphism on fundamental groups induced by the inclusion map is one-to-one.

$$i : A \to X$$

$$i_* : \pi_1(A, x_0) \to \pi_1(X, x_0) \text{ is } 1\text{-}1.$$

Lemma 55.2 There is no retraction of B^2 onto S^1



$$r : B^2 \to S^1 \text{ s.t. } r|_{S^1} = id_{S^1}$$

Lemma 55.3 Let $h : S^1 \to X$ be a continuous map. Then the following are equivalent

1) h is hull homotopic
2) h extends to a continuous map $k : B^2 \to X$
3) h_* is the trivial homomorphism.

Examples

1) The identity map $id: S^1 \to S^1$ is <u>not</u> null-homotopic.

By Th^m (52.4), since $id: S^1 \to S^1$ is the identity map,

then $id_*: \pi_1(S^1, x_0) \to \pi_1(S^1, x_0)$ is the identity.

Since id_* is <u>not</u> the trivial homomorphism, then, by Th^m (55.3), id is <u>not</u> null homotopic.

2) The inclution map $i: S^1 \to \mathbb{R}^2 - \{\vec{0}\}$ is <u>not</u> null homotopic.

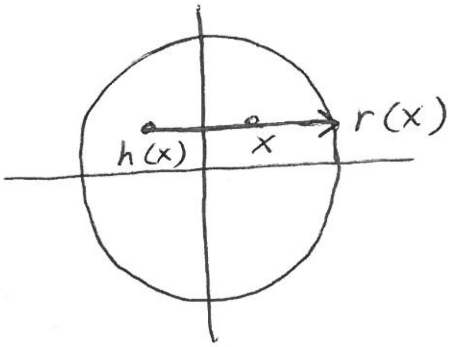
Note $r: \mathbb{R}^2 - \{\vec{0}\} \to S^1$ given by $r(x) = \frac{\vec{x}}{\|\vec{x}\|}$ is a retraction. Hence, by 55.1, $i_*: \pi_1(S^1, x_0) \to \pi_1(\mathbb{R}^2 - \{\vec{0}\}, x_0)$ is one-to-one. Hence i_* is <u>not</u> the trivial homomorphism and, by 55.3, i is <u>not</u> null-homotopic.

Hatcher time

Th^m (1.9) Every continuous map $h: D^2 \to D^2$ has a fixed point x with $h(x) = x$.

Pf Recall $D^2 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$.

Suppose, to form a contradiction that there is a continuous map $h: D^2 \to D^2$ with <u>no</u> fixed point.



Let $r(x)$ denote the ~~ray~~ point on S^1 that meets the ray with end point $h(x)$ and containing x.

Note that since $h(x) \neq x$ for all x, $r: D^2 \to S^1$ is well defined.

Claim: $r: D^2 \to S^1$ is continuous.

Pf| H.W.

Note: If $x \in S^1$, $r(x) = x$. Hence r is a retraction of D^2 onto S^1. This contradicts 55.2. $\square$