

## MATH 555: INTRODUCTION TO 3-MANIFOLDS, HOMEWORK 2

### TOPOLOGICAL MANIFOLDS AND SMOOTH MANIFOLDS

**Due Thursday, 2/23**

Problems (to turn in).

- (1) Let  $S^2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}$  and let  $f : \mathbb{R}^3 \rightarrow \mathbb{R}$  be the map  $f(x, y, z) = z$ . Assume that  $S^2$  is a smooth 2-manifold and that  $f$  is a smooth map. Carefully show that the restriction of  $f$  to  $S^2$  is a Morse function. (Hint: you may use Remark 1.2.13 without proof.)
- (2) Let  $M$  be a topological  $m$ -manifold and let  $N$  be a topological  $n$ -manifold. Carefully show that  $M \times N$  is a topological  $(n + m)$ -manifold.
- (3) Exercise 3 on page 10
- (4) Exercise 4 on page 10