MATH 555: KNOT THEORY, HOMEWORK 1

EQUIVALENCE OF KNOTS AND KNOT DIAGRAMS

Due by Firday, Feb. 1st at 10 am

1. Homework Policy

You are strongly encouraged to work in groups to exchange ideas and help each other understand how to approach problems, but the work you turn in must be your own! If you work with others on an assignment, be sure to indicate the names of the other students on your homework. Additionally, if you use any outside resources (i.e. internet sources, other mathematicians, other books) to help you solve homework problems, you must cite your sources. Failure to follow these rules will result in a score of zero on an assignment and may constitute a violation of academic integrity.

Homework must be legible, well-organized, and written in complete sentences. Handwritten work is fine, but you are encouraged to type up the problems in LaTeX.

2. Readings and Responses.

(1) Read Sections 2.1, 2.2 and 2.3 of “Knots Knotes” by Justin Roberts.

3. Problems

(1) Draw a sequence of figures illustrating that the trefoil can be deformed so that its (non-regular) projection has exactly one multiple point. Briefly argue that every knot can be deformed so that its (non-regular) projection has exactly one multiple point. What is the appropriate analogue of *regular projection* for these one-multiple-point projections? Define an interesting knot invariant based on these one-multiple-point projections.

(2) Using the proof provided in class for the equivalence of smooth knots under scaling as a model, show that if $L : \mathbb{R}^3 \to \mathbb{R}^3$ is a translation of the form $L(x) = x + b$ for fixed vector b and $k(t)$ is a smooth knot, then $L(k(t))$ is smoothly ambient isotopic to $k(t)$.

(3) Read page 5 of “Knots Knotes.” Prove that the crossing number of a trefoil is three.