MATH 590: GRADUATE KNOT THEORY, HOMEWORK 1

EQUIVALENCE OF KNOTS

Due in my office by midnight, Tuesday, 9/6

Problems (to turn in).

(1) Suppose a polygonal knot lies in a plane and bounds a convex region in the plane. (Convex means that any segment with endpoints in the region is entirely contained in the region.) Prove that the knot is equivalent to a knot with three vertices.

(2) Assume the following lemma which is a generalization of a lemma proved in class

Lemma: Given a polygonal knot K, there is a positive constant ϵ_K such that if every vertex of K is moved a distance less than ϵ_K, then the resulting knot is equivalent to K.

Use the above lemma to prove that if $L : \mathbb{R}^3 \to \mathbb{R}^3$ is a translation of the form $L(x) = x + b$ for fixed vector b and K is a polygonal knot, then $L(K)$ is equivalent to K.

(3) Show that every polygonal knot with exactly four vertices is unknotted. (Unknotted means equivalent to a knot with 3 vertices)

(4) Using the proof provided in class for the equivalence of smooth knots under scaling as a model, show that if $L : \mathbb{R}^3 \to \mathbb{R}^3$ is a translation of the form $L(x) = x + b$ for fixed vector b and $k(t)$ is a smooth knot, then $L(k(t))$ is equivalent to $k(t)$.