

## MATH 590: KNOT THEORY, HOMEWORK 3

### CLASSICAL INTEGER INVARIANTS AND THE KAUFFMAN BRACKET

**Due Tuesday, 10/4**

Problems (to turn in).

- (1) Prove that the Kauffman bracket polynomial is invariant under *RIII* Reidemeister moves.
- (2) Prove that the crossing number of a trefoil knot is 3. (Hint: Prove that any knot diagram with 2 or fewer crossings is a diagram of the unknot)
- (3) If  $K$  and  $J$  have bridge number 2, prove that  $K\#J$  has width 14. (Hint: Use Schubert's theorem to deduce the minimal number of maxima and minima for  $K\#J$ . What are the possible heights of these maxima and minima?)
- (4) Find a formula for the Kauffman bracket of a connected sum (i.e.  $\langle K_1\#K_2 \rangle$ ) in terms of the Kauffman brackets of the summands (i.e.  $\langle K_1 \rangle$  and  $\langle K_2 \rangle$ ) and carefully prove your formula holds.