MATH 590: KNOT THEORY, HOMEWORK 4

ADDITIVITY OF CROSSING NUMBER AND MANIFOLDS

Thursday, 10/20

Problems (to turn in).

(1) Carefully prove that if K_1 and K_2 are alternating knots, then $c(K_1 \# K_2) = c(K_1) + c(K_2)$. (Hint: Make sure to prove that the connected sum of two reduced diagrams is a reduced diagram.)

(2) Carefully prove that the unit 2-sphere in $\mathbb{R}^3$ is homeomorphic to the polyhedral surface that is the union the following eight metric triangles in $\mathbb{R}^3$.

$\triangle(1,0,0)(0,1,0)(0,0,1)$
$\triangle(-1,0,0)(0,1,0)(0,0,1)$
$\triangle(-1,0,0)(0,-1,0)(0,0,1)$
$\triangle(1,0,0)(0,-1,0)(0,0,1)$
$\triangle(1,0,0)(0,1,0)(0,0,-1)$
$\triangle(-1,0,0)(0,1,0)(0,0,-1)$
$\triangle(-1,0,0)(0,-1,0)(0,0,-1)$
$\triangle(1,0,0)(0,-1,0)(0,0,-1)$

(Hint: You may use the following theorem from real analysis: If A is a compact metric subspace of $\mathbb{R}^n$, B is a metric subspace of $\mathbb{R}^m$ and $f : A \to B$ is a continuous bijection, then f is a homeomorphism.)